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Philosophy and Methodology of Sciences

Tomasz Placek

Logic 2

(lecture 2)

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Extensional/intensional connectives

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It is not the case that the Earth is round

Extensional/intensional connectives

Thanks to a connective being extensional, it can be fully characterized by a truth-value table. Here is the truth-table for negation:

p	it is not the case that p
1	0
0	1

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Intensional = non-extensional

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If a reasoning is expressed in a language with extensional connectives only, the content of sentences involved is inessential for validity of that reasoning.

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3. auxiliary symbols: (,).

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- if α is a wff, then $(\sim\alpha)$ is a wff as well, and
- if α and β are wffs, then $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \equiv \beta)$ are wffs as well.

Truth vs. proof

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Connectives of CPL

- conjunction ∧ $p \wedge q$

1 Copernicus was an astronomer.
0 Copernicus was an astronaut.
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p	q	$p \wedge q$
1	1	1
1	0	0
0	1	0
0	0	0

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- disjunction ∨ $p \vee q$

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p	q	$p \vee q$
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1	0	1
0	1	1
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Connectives of CPL

- implication $\rightarrow$ $p \rightarrow q$

If you get a very good grade from the test, we'll go to the movies.

p	q	$p \rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

Connectives of CPL

- equivalence $\equiv$ $p \equiv q$

We'll go to the movies if and only if you'll get a very good grade from your test.

p	q	$p \equiv q$
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$v(\alpha \equiv \beta) = 1$ iff $v(\alpha) = v(\beta)$

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3. A value of a wff depends only on the values of those propositional variables that occur in the wff in question.

Tautologies in CPL

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How to check if a wff is a tautology?

Direct method by a truth-table: see the blackboard.

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Assume indirectly that the formula is not a tautology. Then there is a valuation that assigns 0 to that formula. You may find that this assumption leads to absurdity: it assigns both 0 and 1 to some sub-formula of the initial formula. Having found such absurdity, you learned that the indirect assumption is absurd, and accordingly, there is no valuation that assigns 0 to the formula in question, i.e., this formula must be a tautology.

More in the discussion groups.