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Determinism

(lecture 9 and 10)

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Envoi concerning probabilities

Frequency interpretation, natural and hence attractive for a scientist, fares badly with respect to our math criterion. To recall,

- In a sequence, there exist r(A), r(B) but not r(A and B), which suggests fr. probability is not defined on a field … But the examples are not collectives… Further, fr. prob violate countable additivity…

Subjective interpretation meets nicely the criterion (but watch out for the idealizing assumptions about betting agents in the Dutch books theorems). Yey it is about degrees of beliefs.

Each interpretation of probabilities can be selected for an exam topic.

Literature:

- A. Hajek's entry in http://plato.stanford.edu/entries/probability-interpret/

- P. Maher's notes on von Mises, http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.175.6281&rep=rep1&type=pdf

- A.Hajek "Fifteen Arguments Against Hypothetical Frequentism", http://philrsss.anu.edu.au/people-defaults/alanh/papers/fifteen.pdf

- An entry on Dutch Books in Stanford Encyclopedia of Philosophy, http://plato.stanford.edu/entries/dutch-book/

- A. Hajek SCOTCHING DUTCH BOOKS?, http://www.hss.caltech.edu/~jiji/

Determinism and indeterminism

First elucidation

Block universe: only one future course of events is compatible with the present state of the universe;

Modal: each event in the world has exactly one possible continuation;

Via laws: if any two realizations of the world's laws agree at a time, then they agree always afterwards.

- Is the world deterministic?
- Are our theories deterministic?
- How do agents act in our indeterministic world, if it is indeterministic ?

Two traditions of approaching determinism

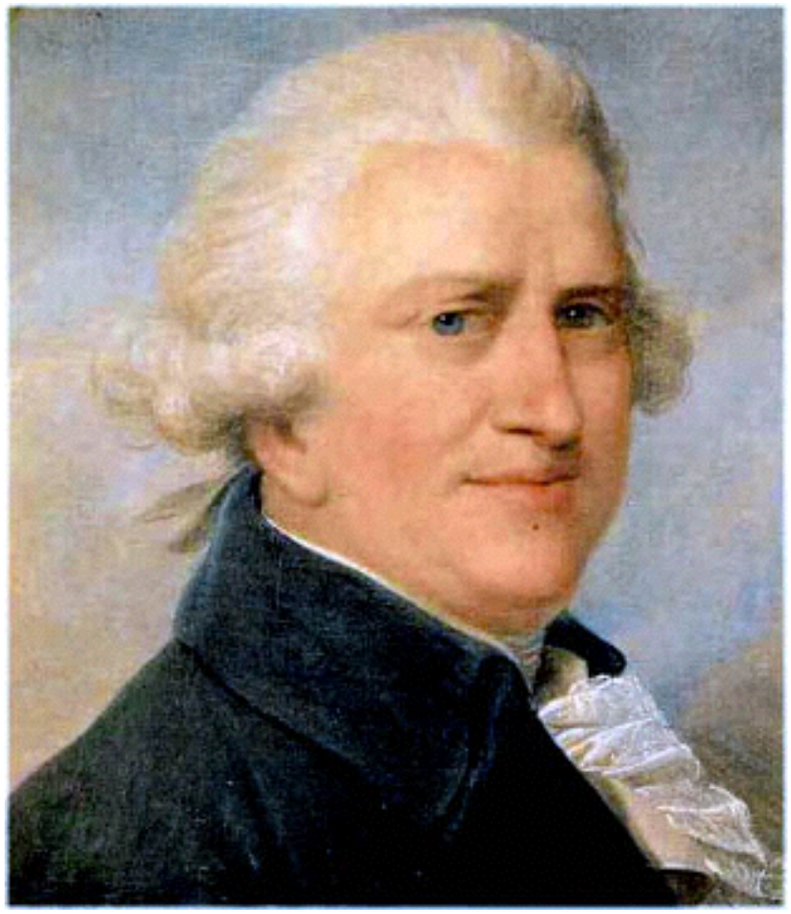
- God's omniscience vs. agents' freedom: if God knows that it will be *p*, no-one can make that it won't be *p*.

Appeal to God unnecessary. God (anyone) knows that p ⇒p is true.

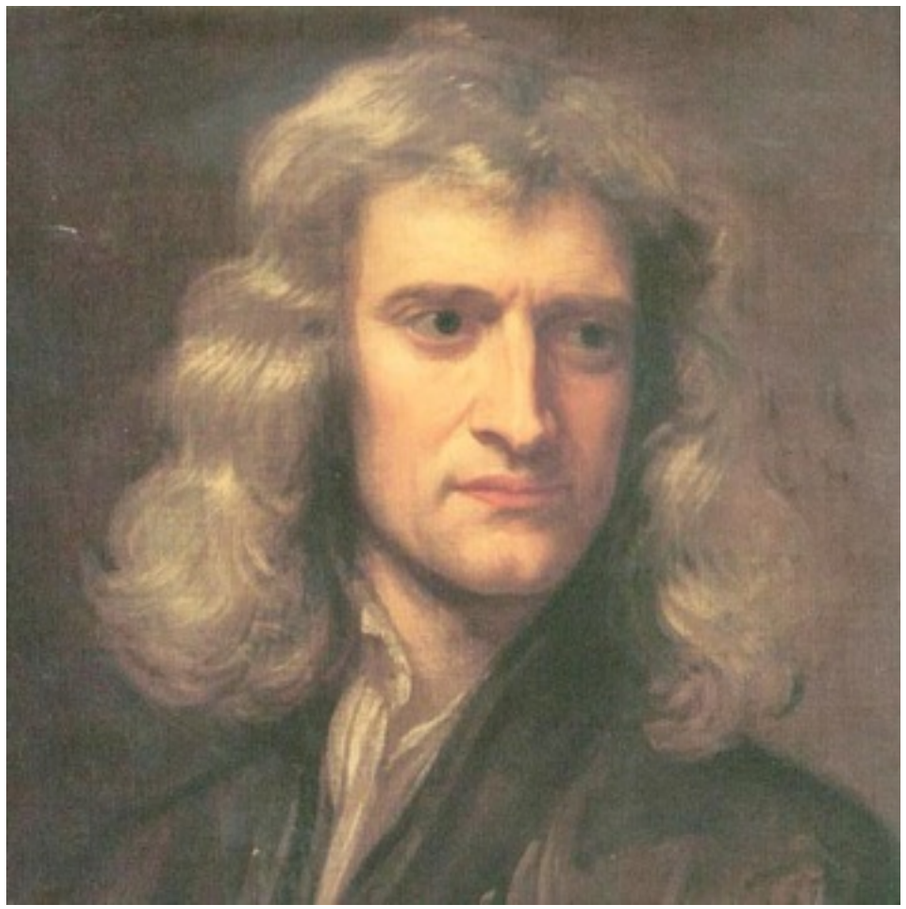
The gist: what is an adequate concept of truth for future tense sentences that allows for indeterminism (free actions)

- Laplacean determinism (physics-motivated)

Modern determinism



Pierre-Simon Laplace



Isaac Newton

We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes. The human mind offers, in the perfection which it has been able to give to astronomy, a feeble idea of such an intelligence. (Laplace 1820)

Where does the vision of the Laplacean intelligence come from?

Reflection (not necessarily correct) about Newton's point particle mechanics and celestial mechanics.

All information about a system's mechanical instantaneous properties is coded by numbers describing the system's spatial location and its momenta. If the system consists of N mass points, 3N numbers for locations and 3N numbers for momenta are needed. In total, we have thus 6N numbers. This is called instantaneous state of Newton's point particle mechanics.

Then how does Newton's law F= ma work? If you know force F (i.e., a mathematical formula for it), then, having the data concerning a system's location and its momenta at a given instant (i.e., 6N numbers) one can compute the system's location and momenta (a set of 6N numbers) for an arbitrary (earlier or later) moment of time.

(Red means controversial.)

The workings of Laplacean intelligence

Input:

- knowledge of an instantaneous state of the world (local data)

- knowledge of all forces acting in the world (global data) What does it mean, to know forces?

Output:

- Laplacean intelligence "has before its eyes" the whole future (and past) But "…eyes.." is a metaphor…

First doubt

Even if Laplace's description of Newtonian physics is correct, should other sciences have a similar form?
Today this sounds absurd, but Locke, Hume and others believed in finding, e.g., a psychological force, to be fed into Newton's dynamic law.

Another doubt

...it will be possible to compute ...: a problematic but highly interesting claim. We now have formal concept of computability, believed to capture this intuitive notion. (generalized computers, Church machines, Turing...)

If we allow for any mathematical formula to stand for a force, some resulting dynamical equations will not be computable.

On the other hand, in physics we have encountered only computable formulas for forces. Is this a mere accident, or some deeper mystery?

A somewhat different formulation:

a system's temporal evolution, i.e., a function from time to the system's states, can be given by a non-computable function s(t)

The system's evolution is then not computable, but it is still unique. Intuitively, that's determinism.

Non-computable cases cast shadow on determinism as computability.

Another doubt

... one can calculate the system's state for an arbitrary moment of time....

That's false. A central theorem of differential calculus (Peano) says that, under certain conditions, we can extend a solution around every value of the argument in some (possibly small) neighborhood. One illustration: five planets in Newton gravitation.

This is the problem with extendability to global solution

Yet another doubt (local existence)

... one can calculate the system's state for an arbitrary moment of time....

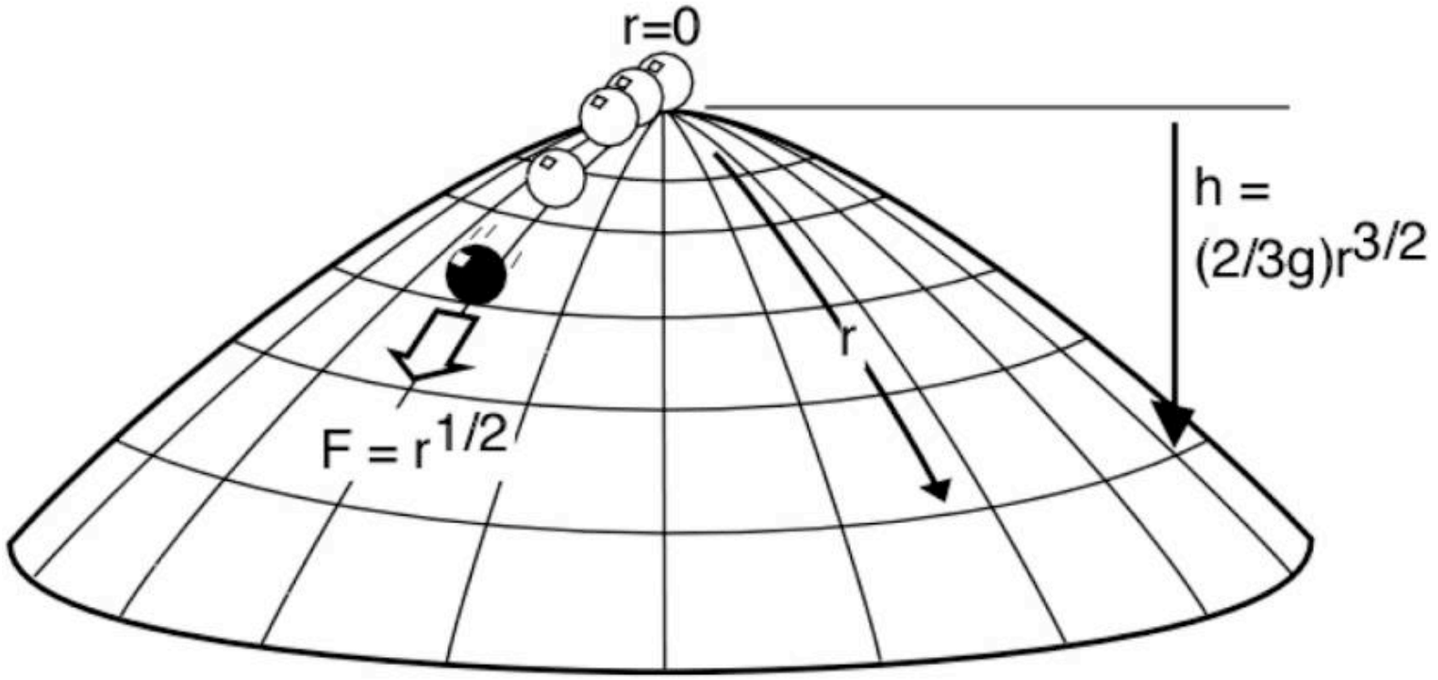
… if Newton's law has unique solutions. A central theorem of differential calculus (Lipschitz) says that, under a certain condition, locally there exists a unique solution for a system of first-order ordinary differential equations. (Newton's law always yields a system of first-order ordinary differential equations.)

under certain condition = formula for force is C1.

An interesting example of failure of determinism in Newtonian physics due to a violation of this condition.

Odd result: Norton’s dome (2008).

A mass point on a (peculiarly shaped) dome



Norton's (2008) dome: essentials

$$h = (2/3g)r^{3/2}$$

$$F = g \sin\alpha = g\ dh/dr$$

$$a = d^2r/dt^2 = g\ dh/dr = r^{1/2}$$

$$d^2r/dt^2 = r^{1/2}$$

Solutions:

$$r(t) = 0$$

$$r'(t) = \begin{cases} (1/144)(t-T)^4 & \text{for } t \geq T \\ 0 & \text{for } t \leq T. \end{cases}$$

Let us check that the above are solutions.
Let's calculate $d^2r'(t)/dt^2$. We obtain

$$a(t) = \begin{cases} (1/12)(t-T)^2 & \text{for } t \geq T \\ 0 & \text{for } t \leq T. \end{cases}$$

Hence $a(t) = r^{1/2}(t)$

How this case is related to the theorem about the unique existence of solutions to differential equations?

Our case is this $dr/dt = \dot{r} \quad d\dot{r}/dt = f(r)$

The theorem (roughly) says: if f(r) is C1 (has a continuous first derivative) on Δ, then the above equation has a unique solution on Δ.

But $f(r) = r^{1/2}$ is not C1 in a neighborhood of 0

Moral: by allowing for arbitrary forces in Newton's mechanics, we get indeterminism. Recall however that the "strange" force comes from a peculiar shape of the dome. Can we exclude some shapes? We cannot.

Lessons from the Dome:

We haven't said what determinism is (haven't elucidated Laplace's metaphor), yet, we were able to analyze determinism of the Dome.

We will be able to do the same for any theory whose defining laws take the form of differential equations.

Recipe: identify the type of diff. eqs, consult a math book on the existence and uniqueness theorems for this type of diff. eqs.

Important caveat ("surplus of math structure")

What is a relation between states posited by a theory (n-tuples of real numbers, vectors in Hilbert space, tensors on tangent space of a manifold, etc.) and *real* states of a *real* system?

Hard: what are real states?

Example: electric and magnetic fields, E and B or electromagnetic potentials, A and ψ

On the assumption that A, ψ are theoretical, whereas E, B are real, the relation is many-to-one:

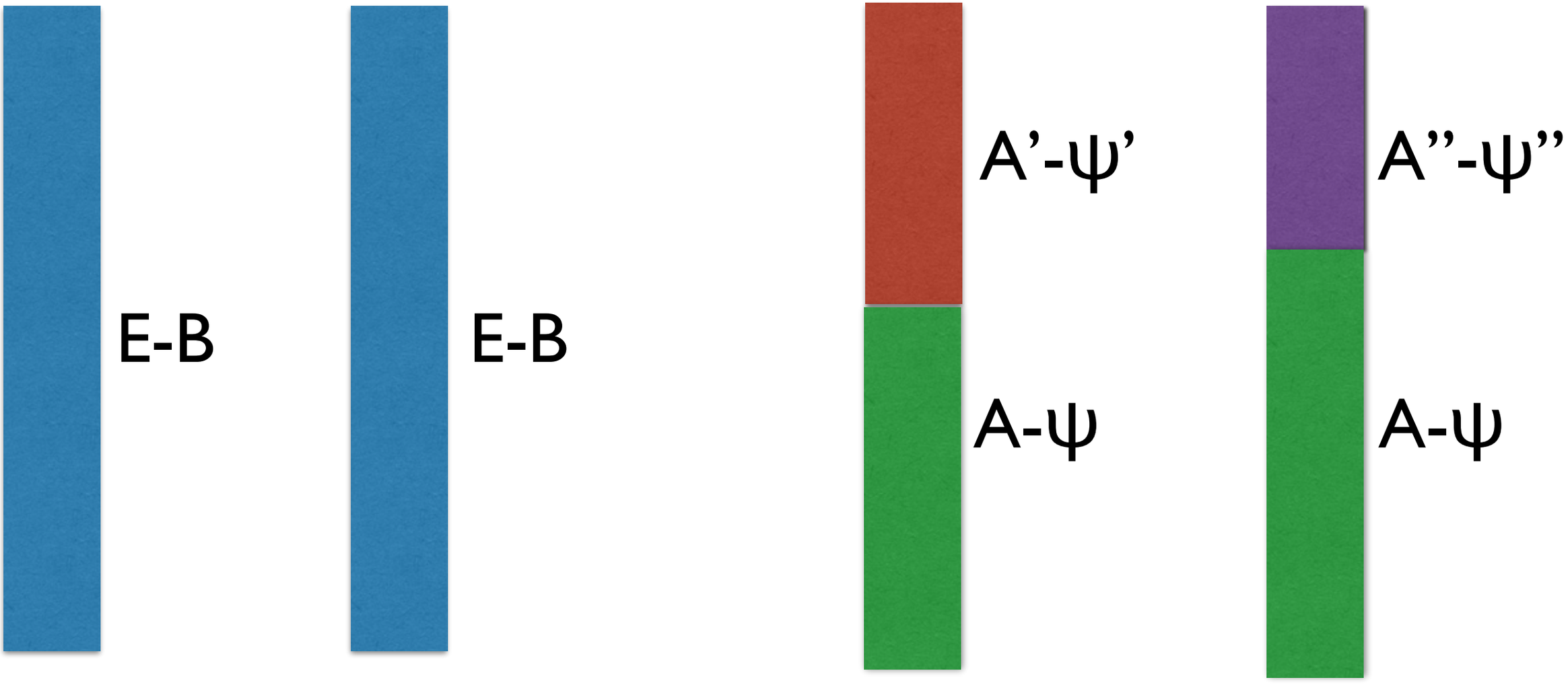
many pairs (A, ψ) yield single pair (E, B)

There is thus some freedom to choose A, ψ to generate a fixed E, B, known as gauge freedom.

$$A' = A + \nabla\psi \qquad \psi' = \psi - \partial\psi/\partial t$$

E-B determinism

A-ψ indeterminism



A simple recipe of approaching determinism via diff eqs is not at all simple.

Hence a need to define/analyse determinism more generally.

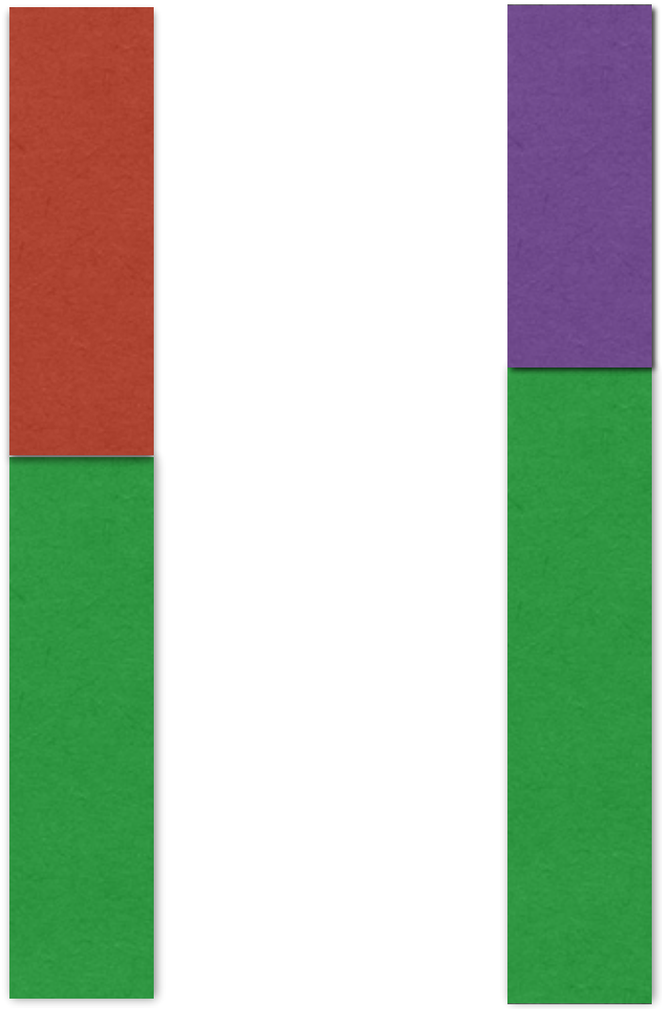
Recall: Laplace had a metaphor, "before its eyes"

Reading it as "can be calculated" inadequate, as the evolution can be governed by a non-computable function

Today's standard: possible-worlds analysis

Possible-worlds account of Lewis (1983).

Two worlds are divergent if they have exactly alike initial segments, followed by different later segments.



"First, a system of laws of nature is Deterministic iff no two divergent worlds both conform perfectly to the laws of that system.

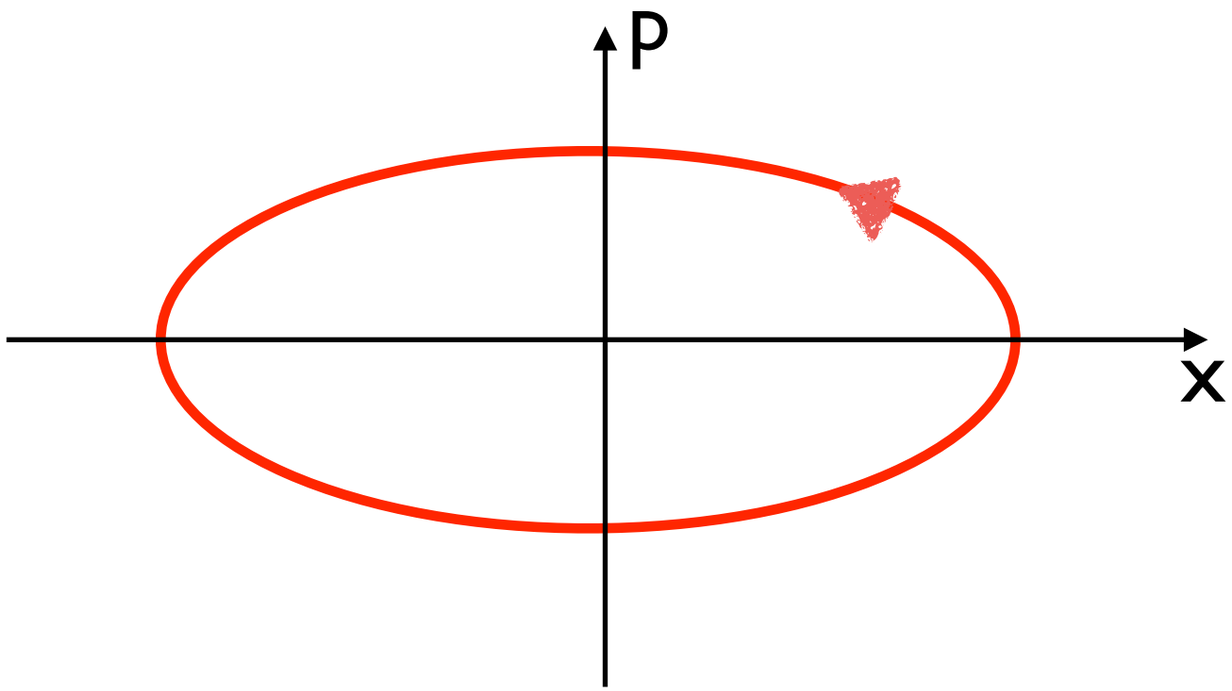
Second, a world is Deterministic iff its laws comprise a Deterministic system.

Third, Determinism is the thesis that our world is Deterministic." (Lewis 1983)

Demystifying possible worlds: how this analysis works?

Is a harmonic oscillator (F= - kx, a model of pendulum) in one dimension deterministic?

Our oscillator is “the possible world” and we ask if it is deterministic. The oscillator is described by this diagram:



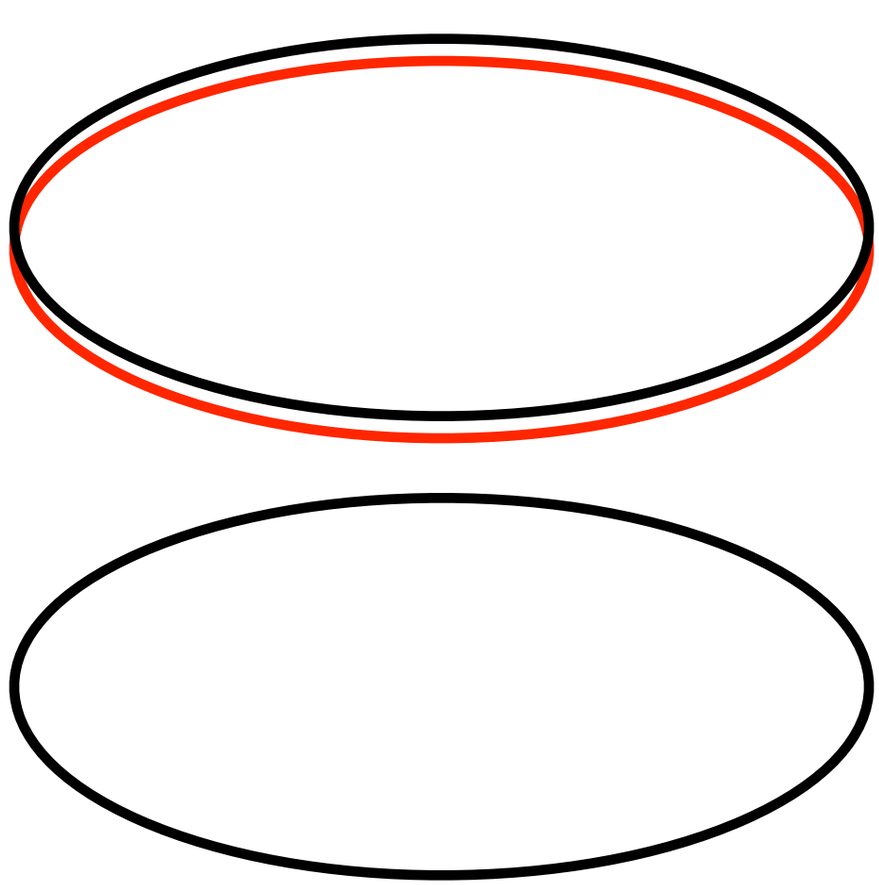
We need next to produce a set of other possible worlds that have a law F = -kx (among other laws) and that could at some time be isomorphic to our oscillator.

Thus, their evolution can be described on plane; these must be other one-dimensional oscillators. How to get "other"? By varying initial conditions (changing how strong it was pushed, for instance)

We landed thus with a set of ellipses, each representing a possible world.

We ask: if two ellipses agree on their initial temporal segments, do they agree later on?

The answer is, of course they do:



Determinism of the theory of evolution

"Bad year for the Tatra bears" - *Trybuna Ludu* reports in 1994

Prerequisites for evolution (Levontin 1978):

1 There is reproduction with some inheritance of traits in the next generation.
2 In each generation, among the inherited traits there is always some variation.
3 The inherited variants differ in their fitness, in their adaptedness to the environment.

Three debates over determinism of the theory of evolution

- argument from rewinding the tape of life is wanting
- percolation argument - from QM to evolution
- autonomous determinism argument - indeterminism or incomplete knowledge?

Chaos (sensitivity to initial conditions) is irrelevant for determinism

Det: A system is deterministic iff

if initial conditions are known, the future evolution can be calculated.

Chaos: unless the initial conditions are known with any degree of precision, the future evolution cannot be calculated (otherwise, it can be calculated)

Two options:

1. *we* cannot know ini con with any degree of precision
2. *idealized agent* can know ini con with any degree of precision

Option I

A system is deterministic iff

if initial conditions are known, the future evolution can be calculated.

The rhs is thus true, so a system is determin.

Option II

A system is deterministic iff

if initial conditions are known, the future evolution can be calculated.

The rhs is thus true, so a system is determin.

Literature (and a guide to exam topics)

In last three decades there has been intensive research in determinism of physical theories, with many surprising discoveries. J. Earman's entry in Elsevier's *Encyclopedia for Philosophy of Physics* is the best survey. His 1986 book is another source. J Butterfield's entry in Routledge Enc of Philosophy is also recommendable.

A general question what determinism is is handled by Lewis 1986. The critics are Belot, Belnap and myself (ask for the papers).

Recommendable papers on determinism of biology / evolution theory are those of Werndl, Stamos, and Millstein (ask for them, please).